PROLOG

- PROLOG is a programming language
- PROLOG: **PROgramming in LOGic**
- Invented by Alain Colmerauer in1973 in Marseille (France)
- Use of **mathematical logic** to represent knowledge
- **Declarative** language

Syntax

- **Constants** are character strings that begin with minuscule letters
  - e.g. `alice`, `edward`, `james_bond_007`
- **Variables** are character strings that begin with a capital or a '_'
  - e.g. `X`, `_G341`, `Stranger`, `The_Older`...
- **Clauses** begin by the positive literals, then the sign ':-' and the sequence of negative literals separated by comma (","); it always end with a dot (".")

```
soeur(X, Y) :- femme( X),
        parents( X, Mere, Pere),
        parents( Y, Mere, Pere).
```

```
soeur( X, Y) v ¬ femme( X) v
  ¬ parents( X, Mere, Pere) v ¬ parents( Y, Mere, Pere).
```

```
femme( X) ∧ parents( X, Mere, Pere) ∧
   parents( Y, Mere, Pere) ⇒ soeur( X, Y)
```

Data Bases in PROLOG

- **Mode "consult"**

```
femme(alice).
femme(victoria).
homme(albert).
homme(edward).
parents(edward, victoria, albert).
parents(alice, victoria, albert).
```

Set of clauses

- **Mode "query"**

```
?- femme(alice).
Yes
?- homme(X).
X = albert
Yes
?- femme(X), parents(X, U, I).
X = alice
U = victoria
I = albert ;
No
```

Atom Conjunction

- **Mode "consult"**

```
femme(alice).
femme(victoria).
homme(albert).
homme(edward).
parents(edward, victoria, albert).
parents(alice, victoria, albert).
soeur( X, Y) :- femme( X), parents( X, Mere, Pere),
        parents( Y, Mere, Pere).
```

Set of <u>Horn clauses</u>

- **Mode "query"**

```
?- soeur(U, edward).
U = alice ;
No
?- soeur(U, V).
U = alice
V = edward ;
U = alice
V = alice ;
No
```

Deduction in PROLOG

Atom conjunction

Positive Horn Clauses

Definition: a **positive Horn clause** is a clause (i.e. A disjunction of literals) that owns one and only one positive literal

Examples:

```
femme(victoria).
homme(edward).
parents(alice, victoria, albert).
soeur( X, Y) :- femme(X), parents(X, Mere, Pere),
  parents( Y, Mere, Pere).
```

Are Horn clauses

`¬barber(X) v shave(X, Y) v shave(Y, Y)` which means: *"the barbers shave those who don't shave by themselves"* is not a Horn clause

`¬barber(X) v ¬shave(X, Y) v ¬shave(Y, Y)` which means *"there is no barber who shaves someone who shaves himself"* is not a positive Horn clause

Declarative programming

- Description of predicate properties:

```
nombre(0).
nombre(s(X)) :- nombre(X).
addition(0, X, X).
addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
```

- Call in query mode:

```
?- nombre(s(s(0))).
Yes
?- addition(s(s(0)), s(s(s(0))), Z).
Z = s(s(s(s(s(0)))))
Yes
```

Output

Result

- **Mode "consult"**

Set of Horn clauses = <u>Rules</u>

```
nombre(0).  nombre(0) →
nombre(s(X)) :- nombre(X).     nombre(s(X)) → nombre(x)
addition(0, X, X).  addition(0,X,X) →
addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
      addition(s(X), Y, s(Z)) → addition(X, Y, Z)
```

- **Mode "query"**

Atom conjunction

```
?- nombre(s(s(s(0)))).
?- addition(s(s(0)), s(s(s(0))), X).
```

- **Work of the Interpreter**

The query is rewritten using rules until it becomes empty.

PROLOG for "babies"

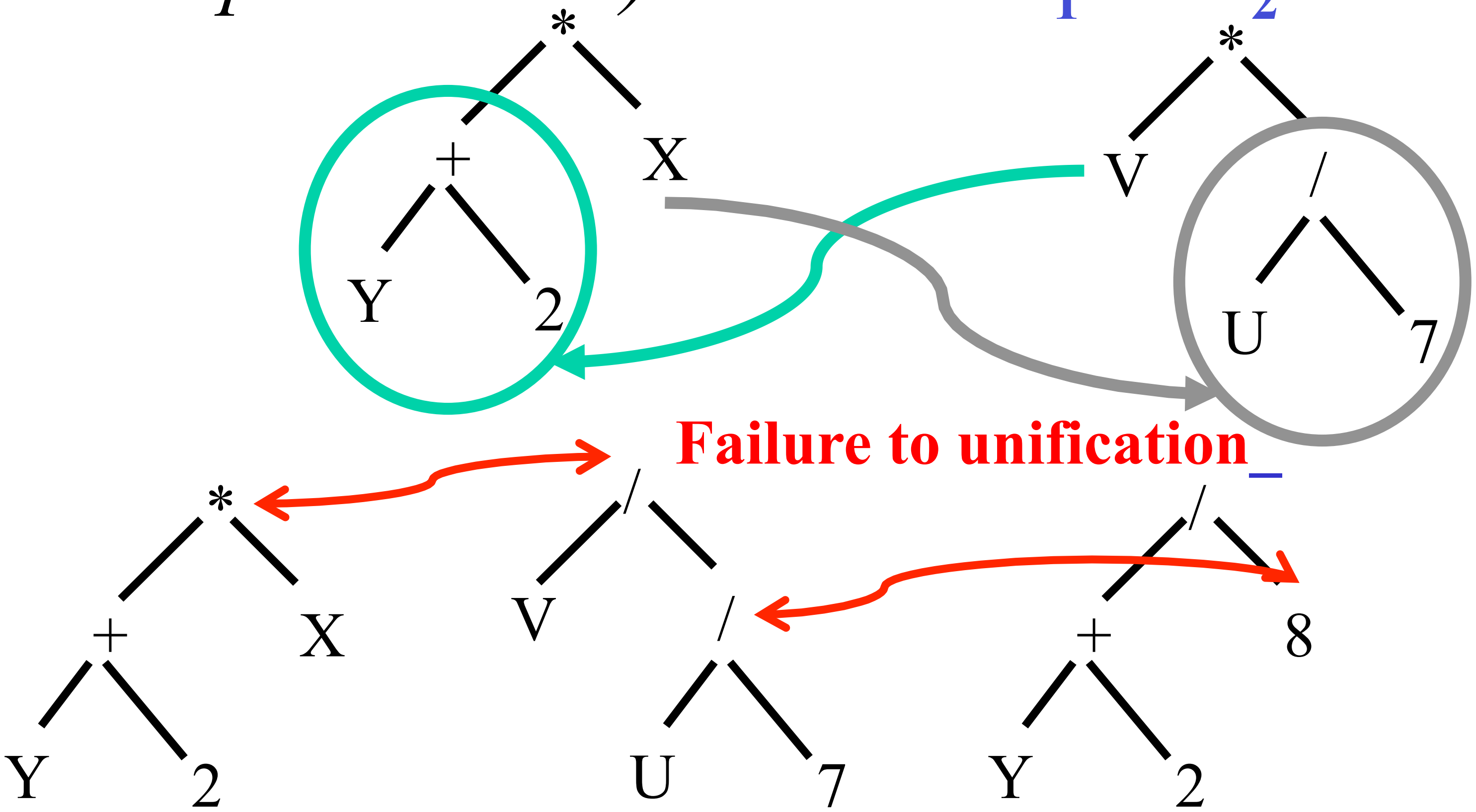
PROLOG strategy

- PROLOG try to apply the "rules" (i.e. the clause) on the atoms of the "query"
- PROLOG makes use of the "unification" mechanism
- The atoms of the "query" are solved sequentially, from left to right.
- The clauses are considered one after one, from the first until the last.

Unification

Unify: to make unique

Unification: two terms t_1 and t_2 unify if and only if there exists a substitution σ *(i.e. a list of variable replacements)* such that: $t_1\sigma = t_2\sigma$



Example: addition

```
:- addition(s(s(0)), s(s(s(0))), Z)        C1    C2
:- addition(s(0), s(s(s(0))), Z1)          C1    C2
:- addition(0, s(s(s(0))), Z2)             C1    C2
□
```

```
C1: addition(0, X, X).
C2: addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
```

Function Inversion

- Description of predicate properties:

```
nombre(0).
nombre(s(X)) :- nombre(X).
addition(0, X, X).
addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
```

- Call in query mode:

```
?- addition(s(s(0)), U, s(s(s(s(s(0)))))).
U = s(s(s(0)))
Yes
?- nombre(X).
X = 0 ;
X = s(0) ;
X = s(s(0)) ;
X = s(s(s(0))) ;
X = s(s(s(s(0))))
```

How PROLOG interprets clauses?

```
C1: femme(alice).
C2: femme(victoria).
C3: homme(albert).
C4: homme(edward).
C5: parents(edward, victoria, albert).
C6: parents(alice, victoria, albert).
C7: soeur( X, Y) :- femme( X),
        parents( X, Mere, Pere),
        parents( Y, Mere, Pere).
```

- **Mode "consult"**

Set of Horn clauses = **Rules**

```
nombre(0).  nombre(0) →
nombre(s(X)) :- nombre(X).     nombre(s(X)) → nombre(x)
addition(0, X, X). addition(0,X,X) →
addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
      addition(s(X), Y, s(Z)) → addition(X, Y, Z)
```

- **Mode "query"**

Atom conjunction

```
?- nombre(s(s(s(0)))).
?- addition(s(s(0)), s(s(s(0))), X).
```

- **Work of the Interpreter**

The query is rewritten using rules until it becomes empty.

PROLOG for "babies"

PROLOG strategy

- PROLOG try to apply the “rules” (i.e. the clause) on the atoms of the “query”
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PROLOG semantics

Language semantics

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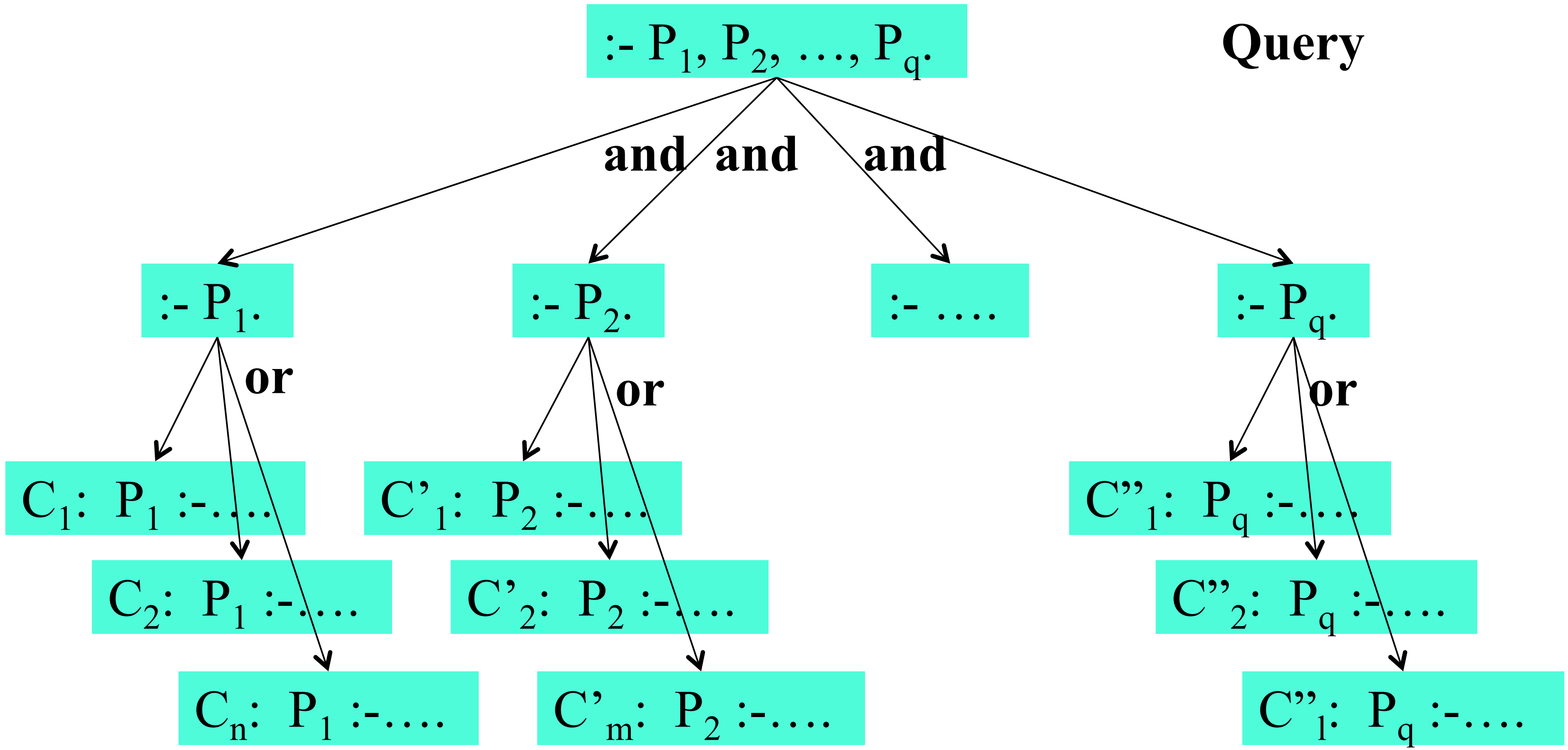
translation in mathematical terms

- Procedural semantics
  - *and/or* trees
  - Derivation trees
- Logic semantics
  - Automatic Proof

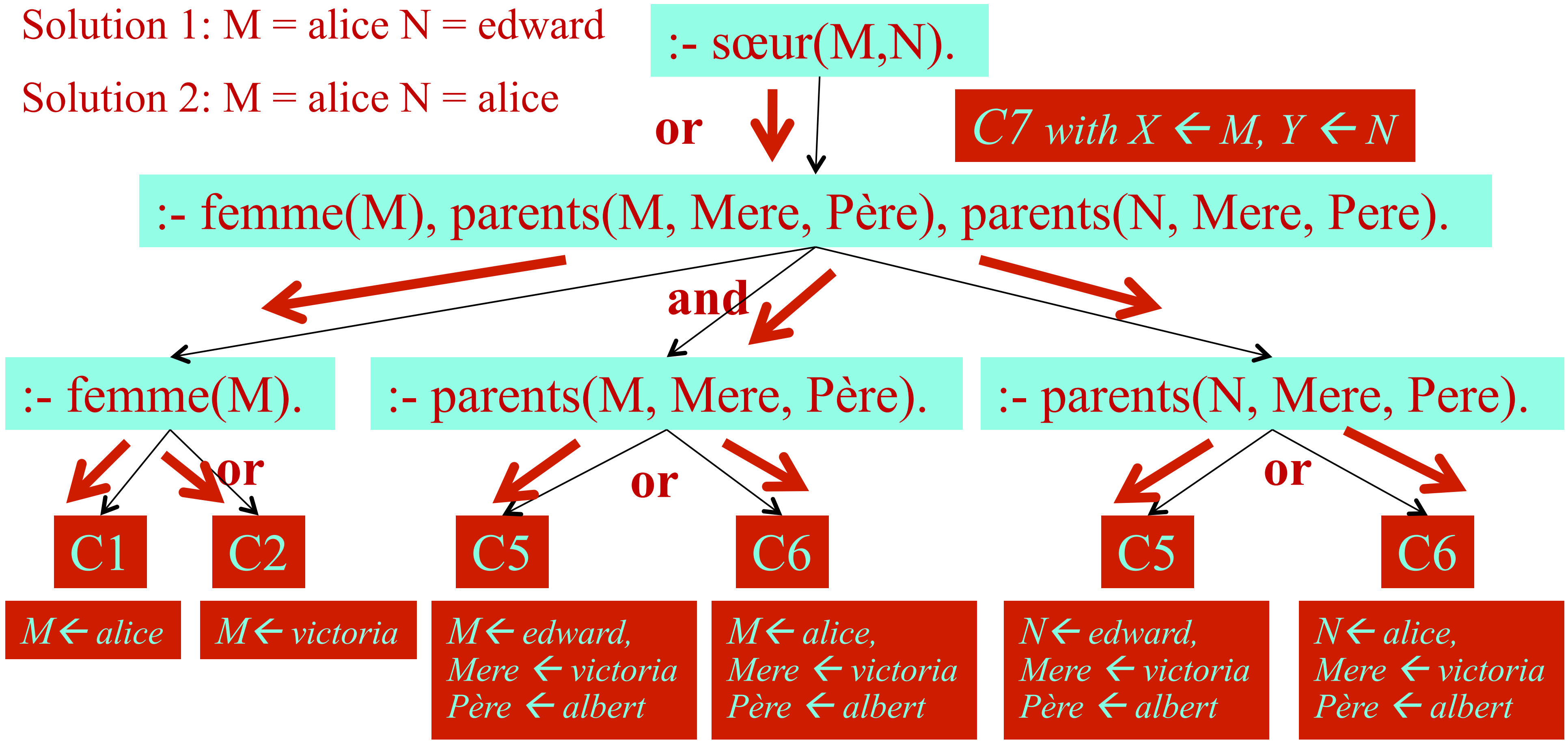
Procedural interpretation: and/or tree



Procedural interpretation: **and/or** trees

Solution 1: M = alice N = edward

Solution 2: M = alice N = alice



C1: femme(alice).

C2: femme(victoria).

C3: homme(albert).

C4: homme(edward).

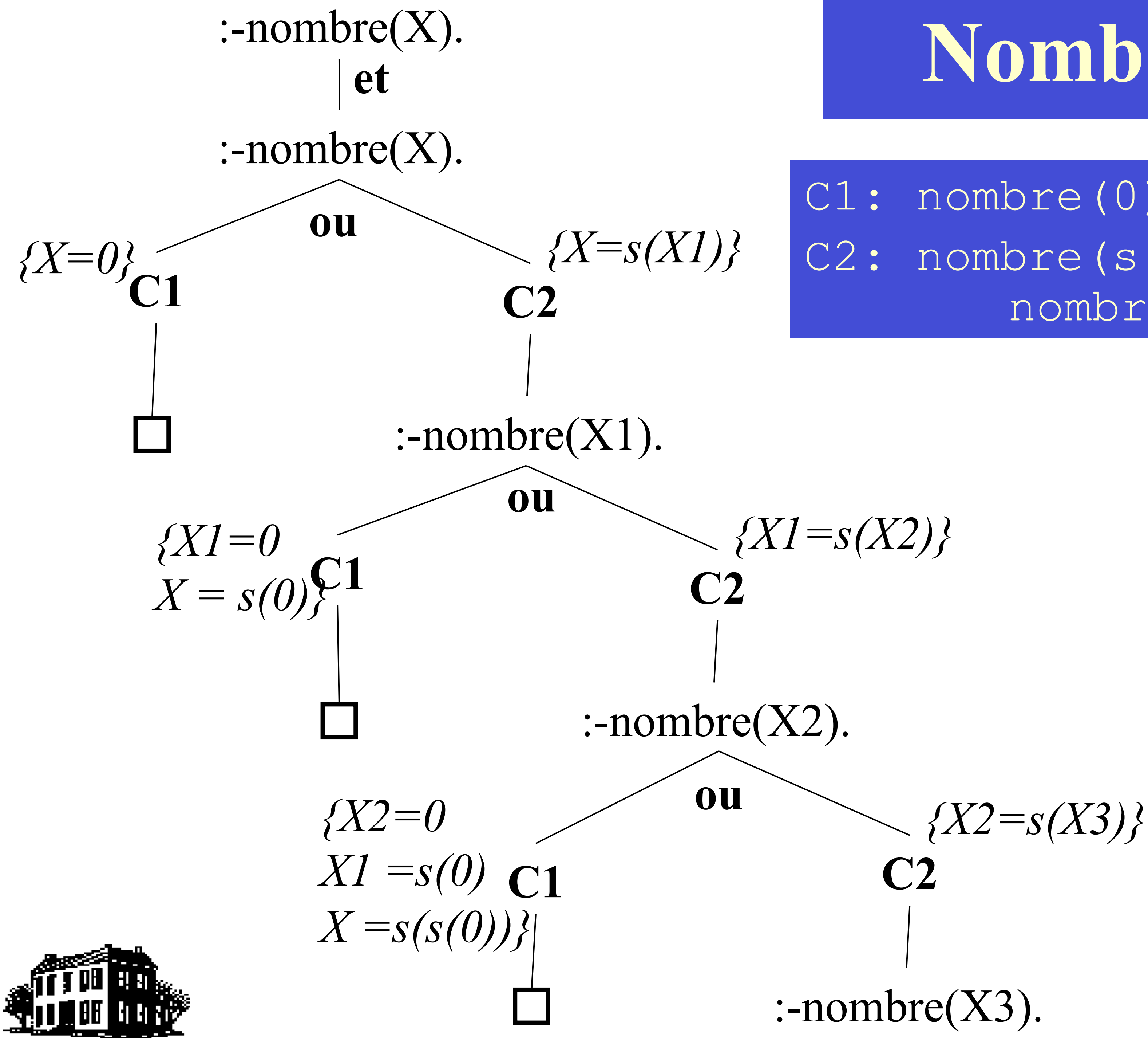
C5: parents(edward, victoria, albert).

C6: parents(alice, victoria, albert).

C7: soeur(X, Y) :- femme(X),
parents(X, Mere, Pere), parents(Y, Mere,
Pere).

Nombre

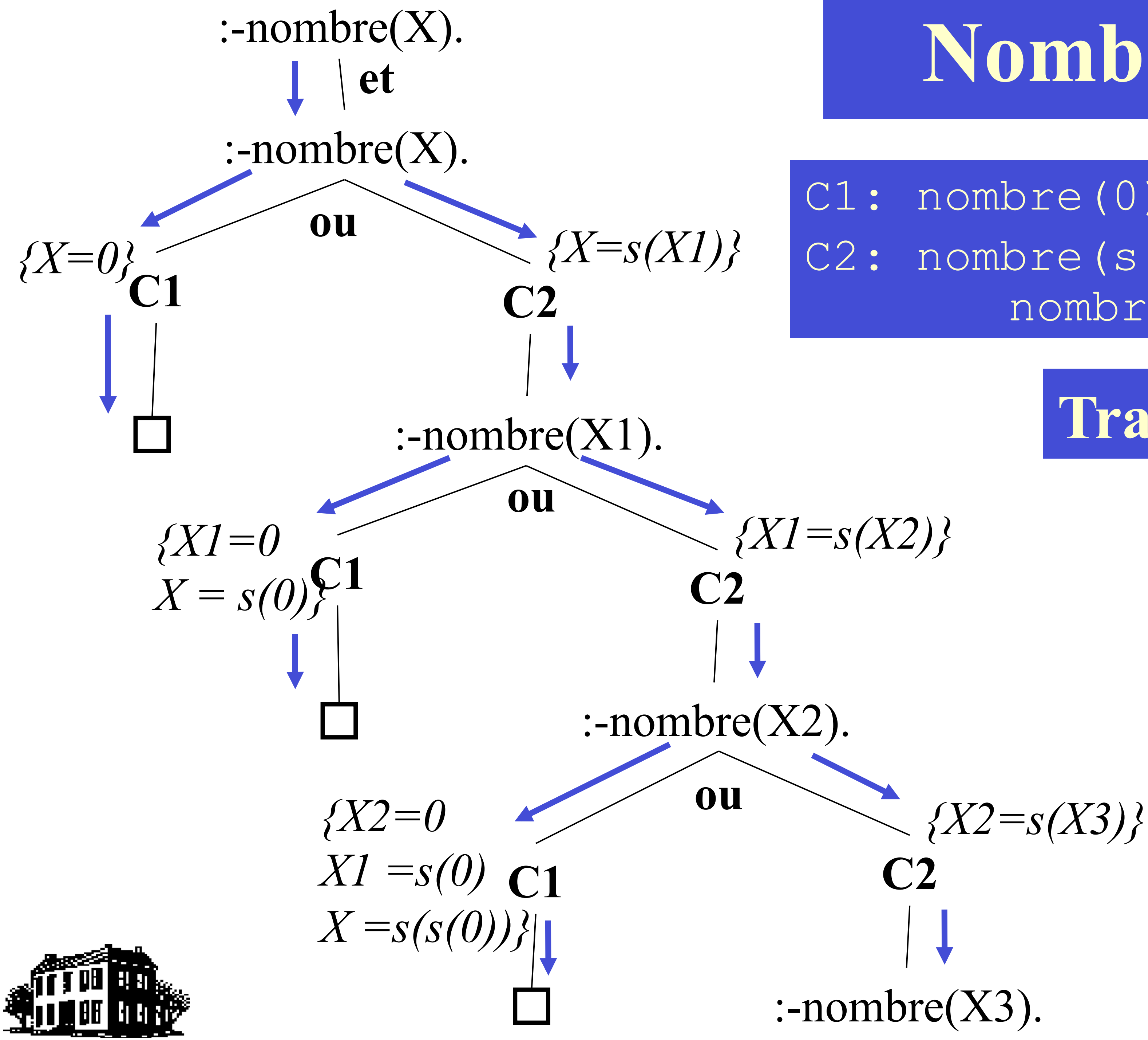
```
C1: nombre(0).
C2: nombre(s(X)) :-
        nombre(X).
```



Nombre

```
C1: nombre(0).
C2: nombre(s(X)) :-
        nombre(X).
```

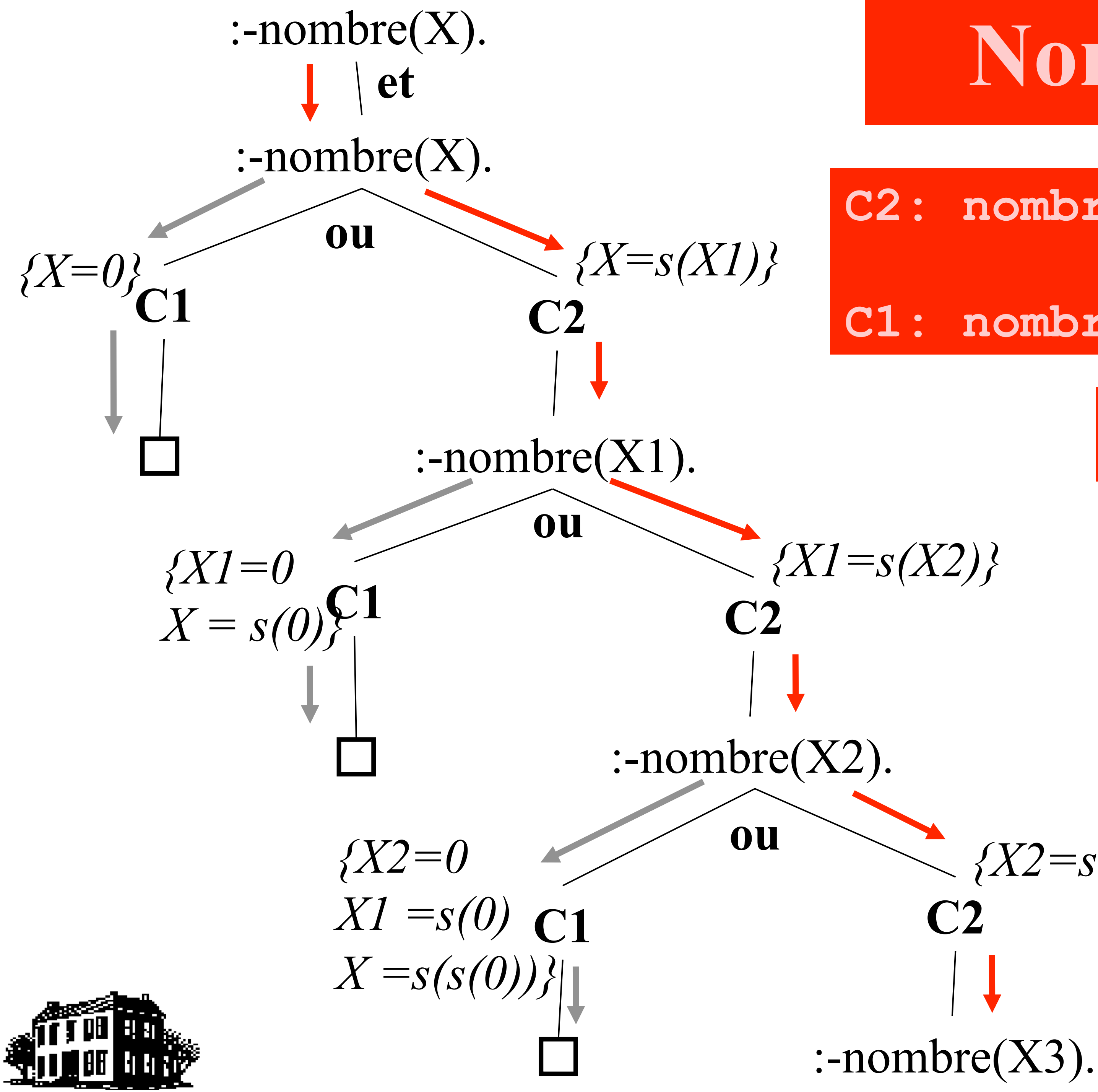
Travel 1



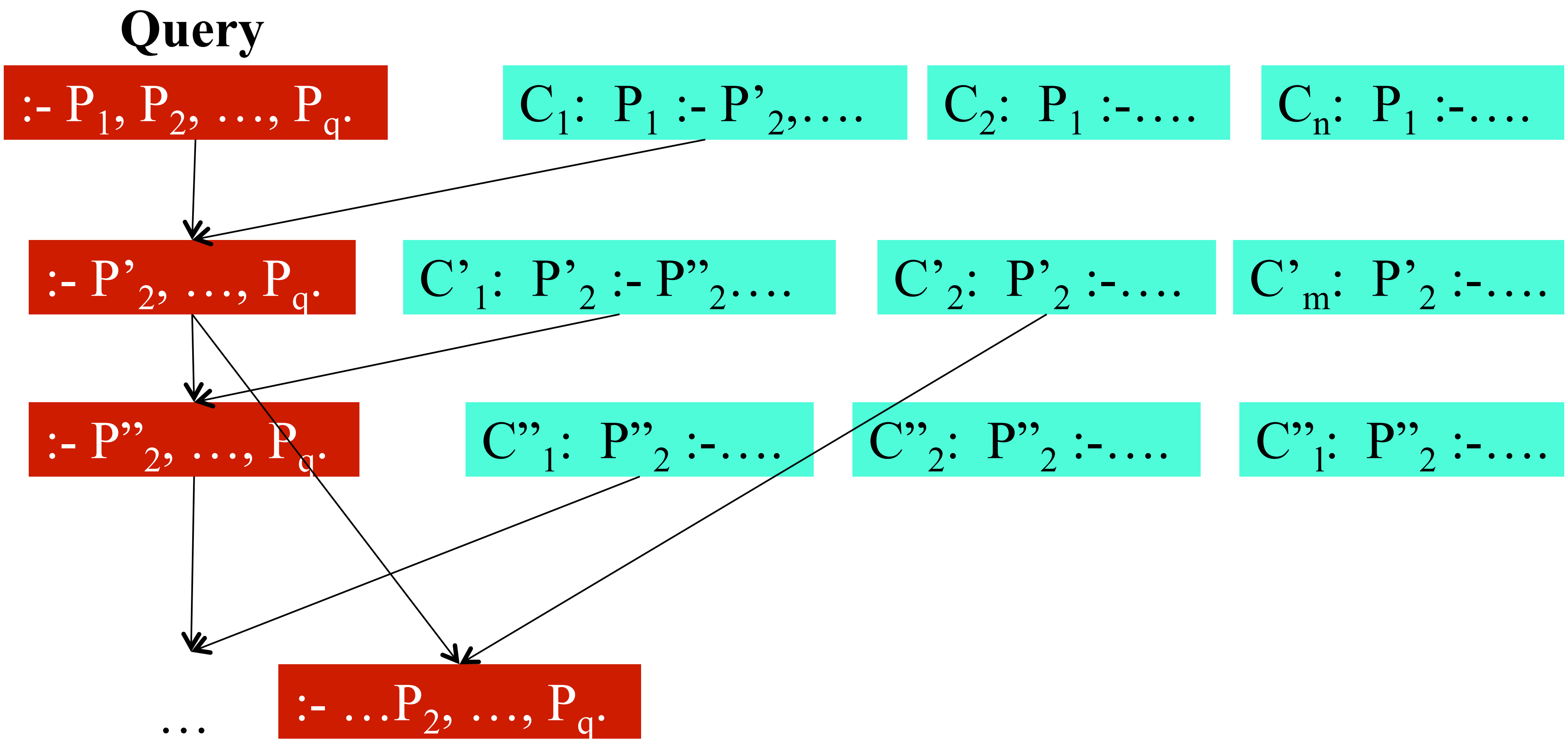
Nombre

```
C2: nombre(s(X)) :-
            nombre(X).
C1: nombre(0).
```

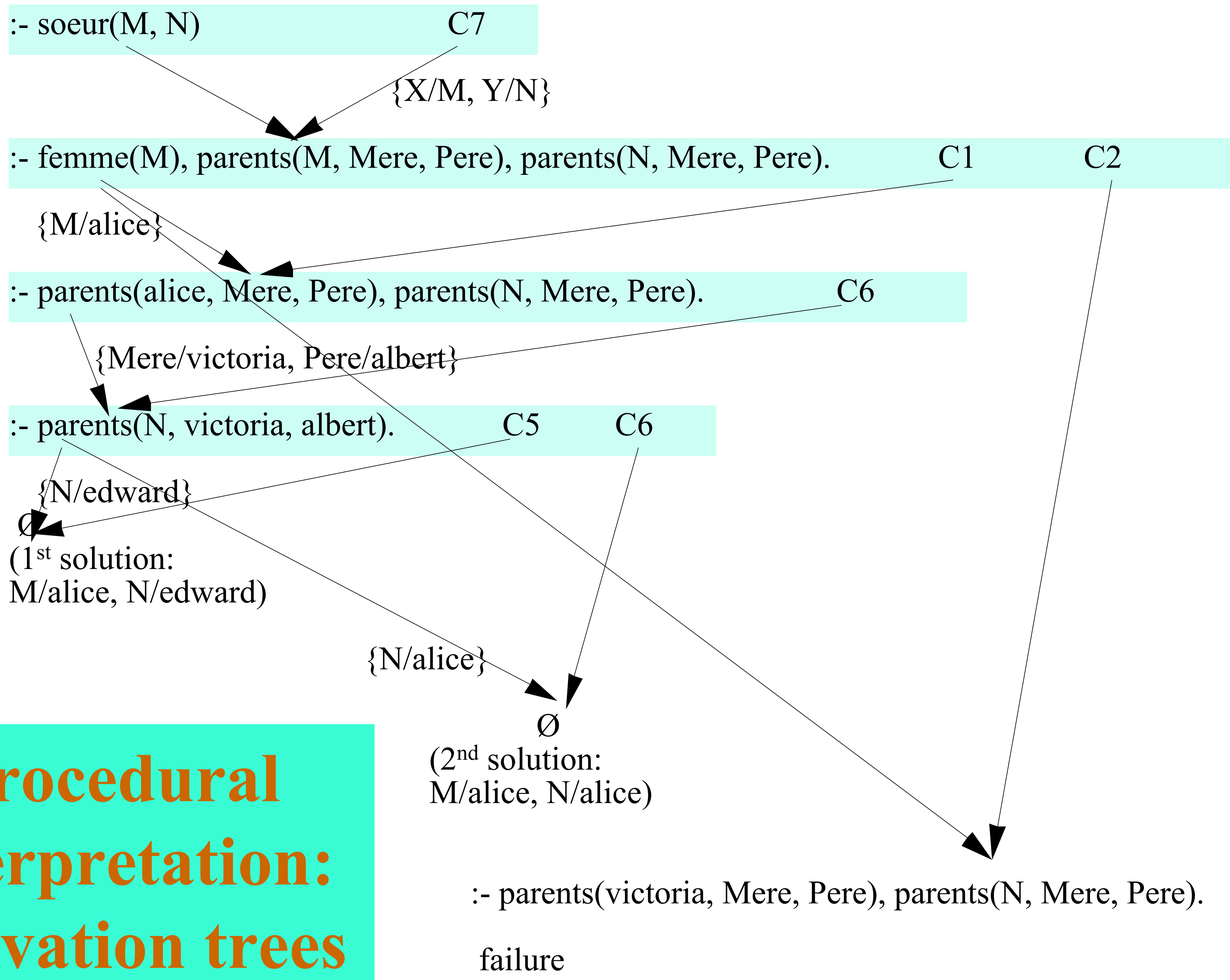
Travel 2



Procedural interpretation: *derivation tree*



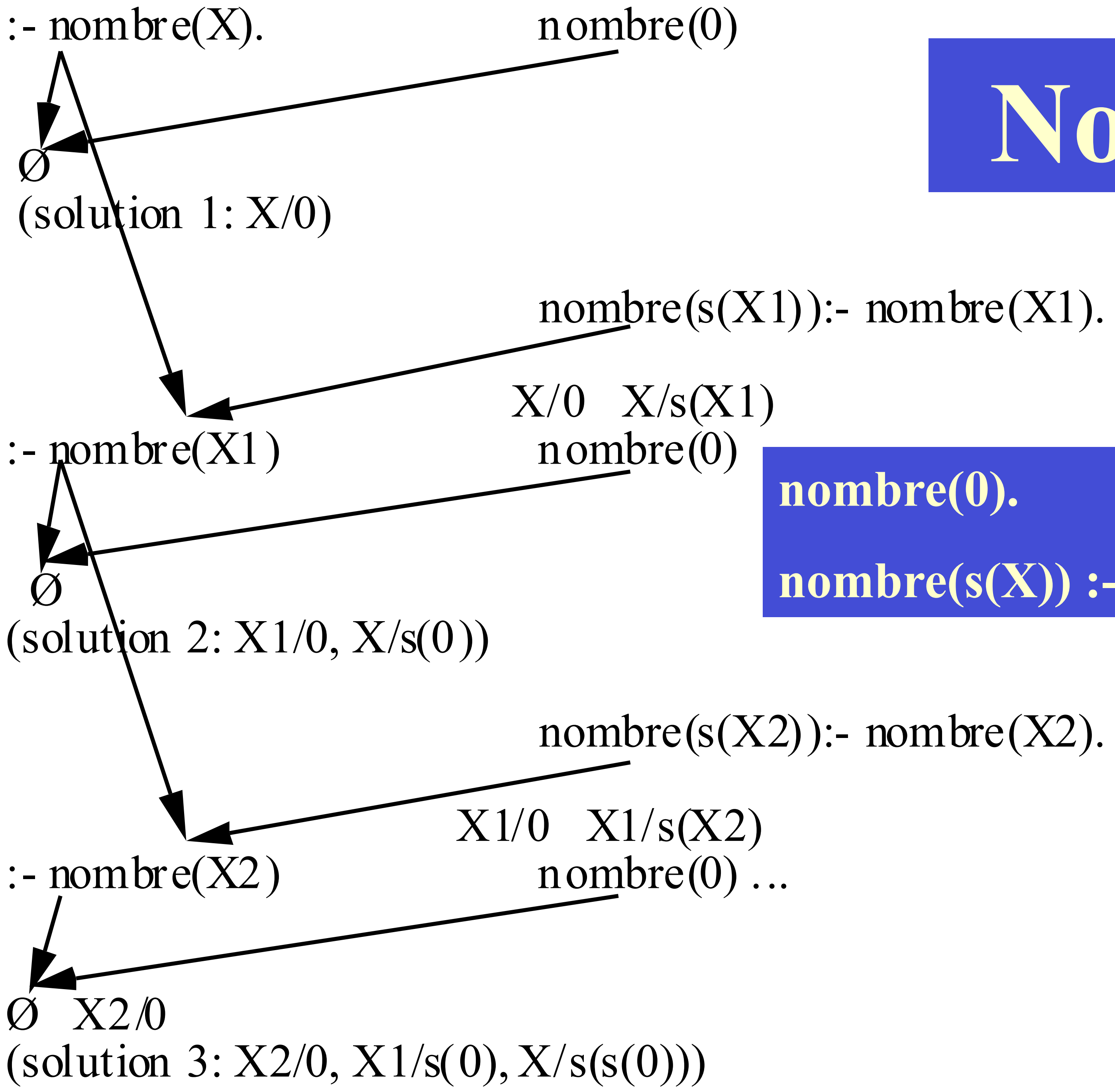
Procedural interpretation: derivation trees



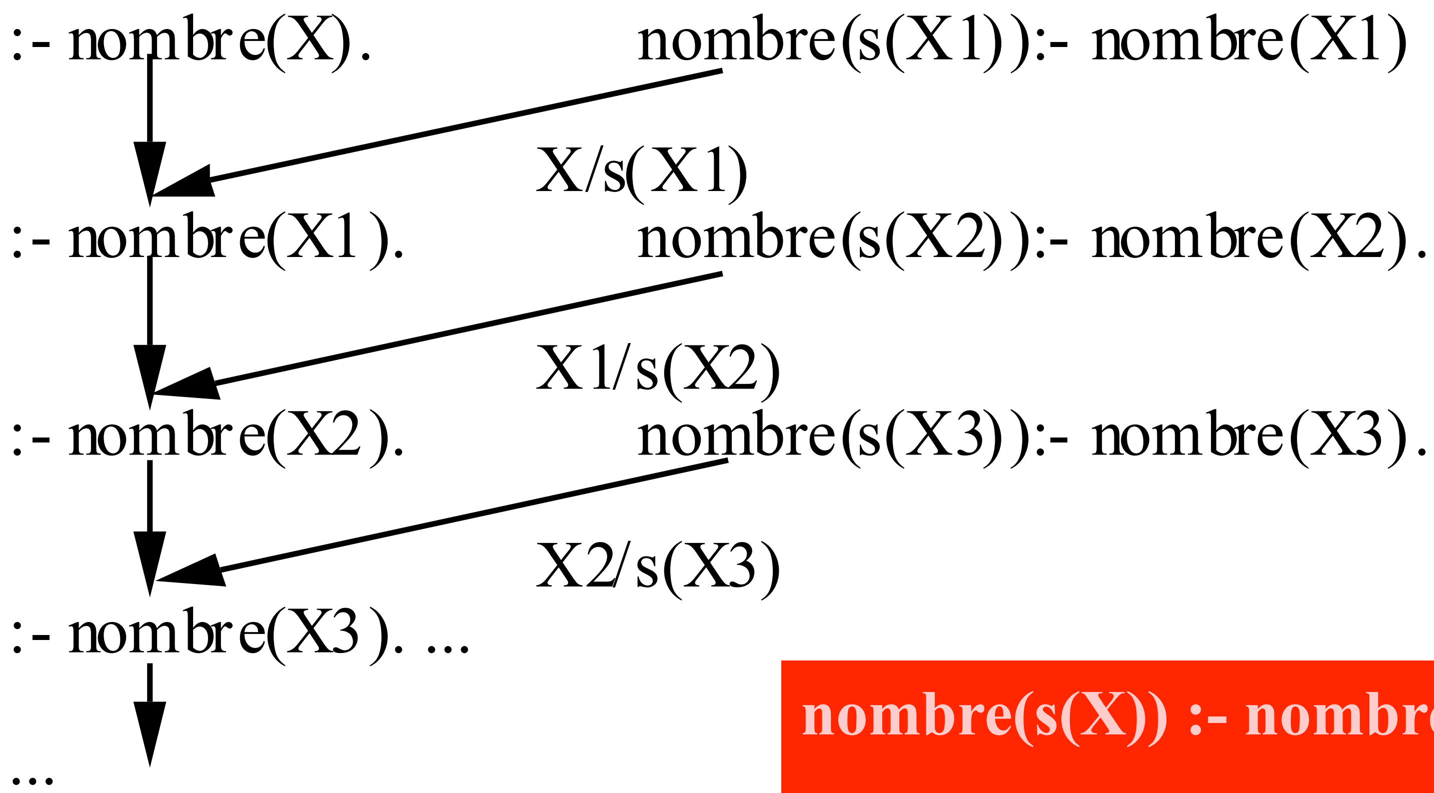
Nombre

nombre(0).

nombre(s(X)) :- nombre(X).



Nombre (2)



nombre(s(X)) :- nombre(X).

nombre(0).

Program

- A program is a set of clauses of which head literal has the same predicate name and the same arity
- Examples
  - Program addition: *[addition(X, Y, Z) means X+Y=Z]*

```
addition(0, X, X).
addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
```

  - Program multiplication: *[multiply(X, Y, Z) means Z=X.Y]*

```
multiply(0, X, 0).
multiply(s(N), P, R) :- multiply(N, P, Q),
      addition(P, Q, R).
```

  - Programme factorial: *[fact_s(N, P) means P = N!]*

```
fact_s(0, s(0)).
fact_s(s(0), s(0)).
fact_s(s(N), R) :- fact_s(N, Q),
         multiply(s(N), Q, R).
```

Procedure call

- A call to a procedure is an atom in the "query" mode:

Examples :

```
?- fact_s(s(s(s(0))), U).
U = s(s(s(s(s(s(0))))))
Yes
?- multiply(s(s(0)), s(s(s(0))), R).
R = s(s(s(s(s(s(0))))))
Yes
?- multiply(s(s(0)), H, s(s(s(s(s(s(0))))))).
H = s(s(s(0)))
Yes
```

Output

Input

Example addition

and/or tree

```
:- addition(s(s(0)), s(s(s(0))), Z)
```

C2 — *{X1 = s(0), Z = s(Z1)}*

```
:- addition(s(0), s(s(s(0))), Z1)
```

C2 — *{X2 = 0, Z = s(Z1) = s(s(Z2))}*

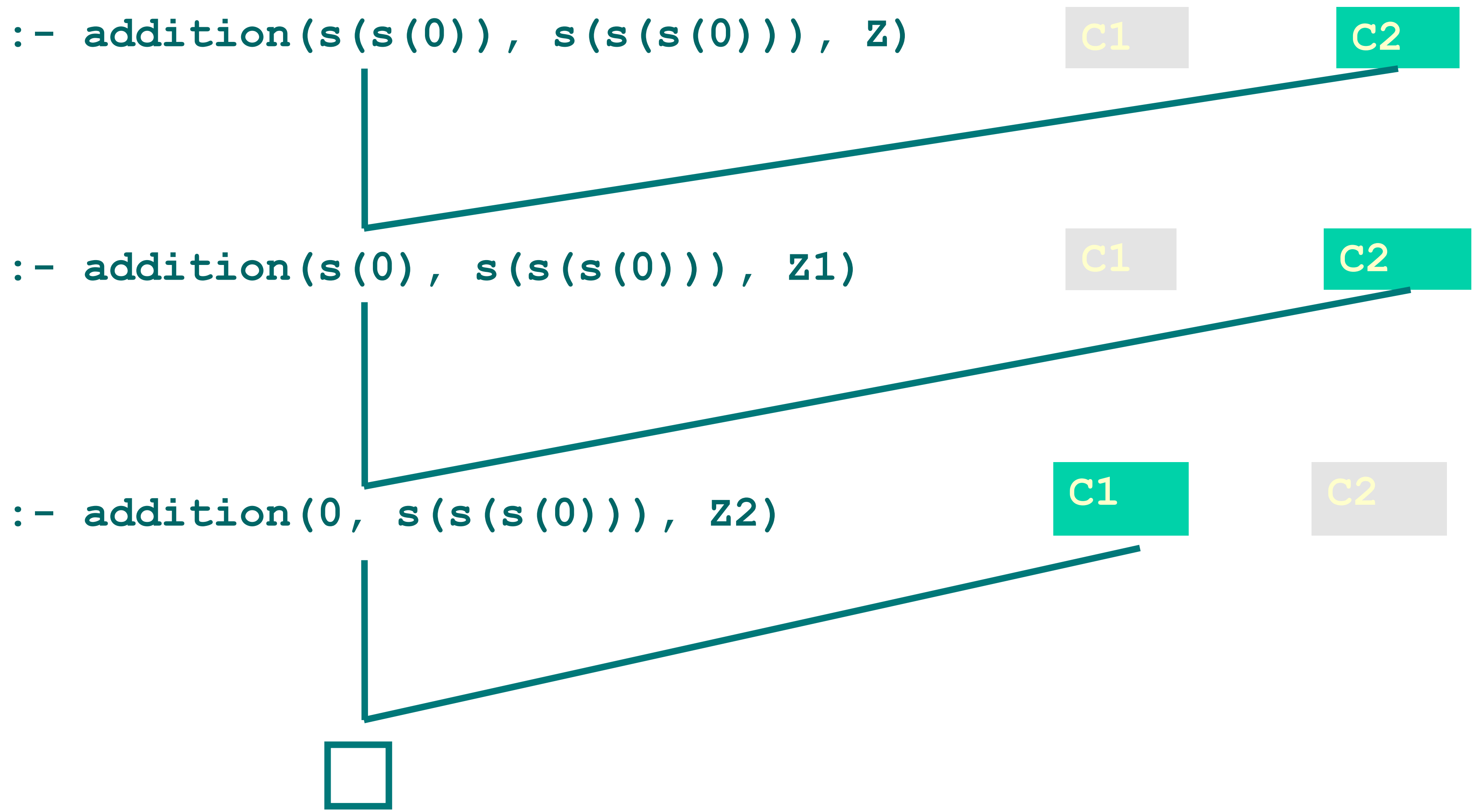
```
:- addition(0, s(s(s(0))), Z2)
```

C1 — *{Z2 = s(s(s(0)))}*

□ ***1st solution:***
Z = s(s(Z2)) = s(s(s(s(s(0)))))

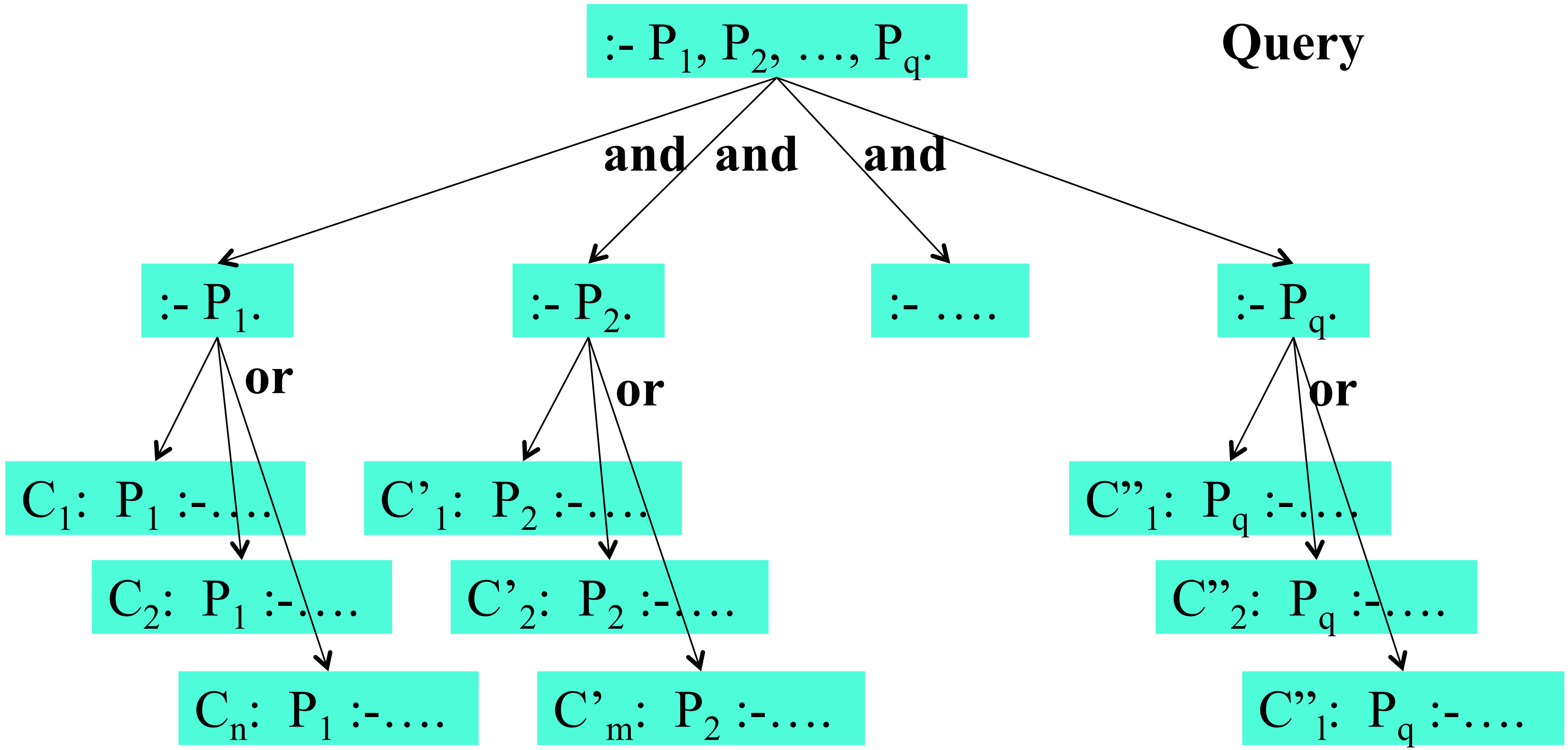
```
C1: addition(0, X, X).
C2: addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
```

Example addition: derivation tree

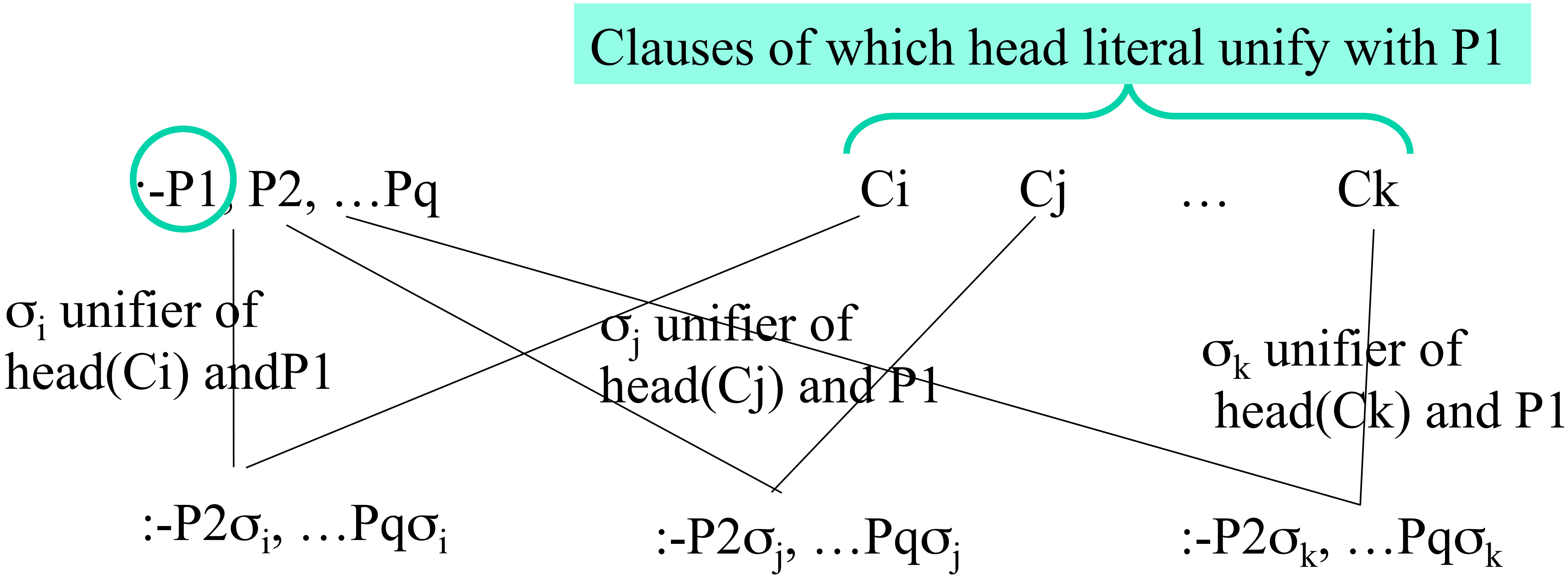



```
C1: addition(0, X, X).
C2: addition(s(X), Y, s(Z)) :- addition(X, Y, Z).
```

Procedural interpretation: and/or tree



Procedural interpretation: derivation tree



Data structures

- Terms:
  - Variables
  - Function with arguments: f(a, g(h, I), K)
- Atoms:
  - Character strings (beginning with a minuscule)
  - Numbers
- Lists:
  - Empty lists, list sequences
- Strings
- Numbers

List structure

- Defined by the *constructor*, cons and by the *empty list*, nil
  - Form: cons(<head>, <queue>)
  - <head> is any literal
  - <queue> is a list

Examples:

- The list (a b c) can be represented by:
  cons(a, cons(b, cons(c, nil)))
- The tree (+ (- b c) (* d e)) can be represented by:

```
cons(+, cons( cons(-, cons(b, cons(c, nil))),
              cons ( cons(*, cons(d, cons(e, nil))),
                     nil),
              nil),
        nil)
```

List structure (2)

- Simplification:
  - The list **(a b c)** is written **[a, b, c]**, which means cons(a, cons(b, cons(c, nil)))
  - In the same way, **(+ (- b c) (* d e))** is written **[+, [-, b, c], [*, d, e]]**
- List constructor: '|'

Example: the list **(a, b, c)** is written:

- `[a, b, c]`
- `[a | [b, c]]`
- `[a | [b | [c |[]]]]`
- `[a, b | [c]]`

Program example

test if an element belong to a list

- Program « appartient » with terms:

```
appartient_t(A, cons(A, B)).
appartient_t(A, cons(B,C)) :-
        appartient_t(A, C).
```

- Program « appartient » with PROLOG representation :

```
appartient0(A, [A|B]).
appartient0(A, [B|C]) :- appartient0(A, C).
```

Execution

```
C1: appartient(X, [X|L]).
C2: appartient(X, [T|L]) :- appartient(X, L).
```

:- appartient(3, [1, 2, 3, 4])

C2

σ_1 = {X1/3, Y1/1, L1/[2, 3, 4]}

:- appartient(3, [2, 3, 4])

C2

σ_2 = {X2/3, Y2/2, L2/[3, 4]}

:- appartient(3, [3, 4])

C1 C2

σ_3 = {X3/3, L3/[4]}

□

Other examples with lists

Concatenation of two lists:

```
concat([], M, M).
concat([A|L], M, [A|N]) :- concat(L, M, N).
```

List inversion:

```
miroirnaif([], []).
miroirnaif([A|Q], R) :- miroirnaif(Q, P),
                concat(P, [A], R).
```

Execution

```
C1: concat([], M, M).
C2: concat([A|L], M, [A|N]) :- concat(L, M, N).
```

```
:- concat([a, b], [c, d, e], R)
```

C2

σ_1 = {A1/a, L1/[b], M1/[c, d, e], R/[A1|N1]}

```
:- concat([b], [c, d, e], N1)
```

C2

σ_2 = {A2/b, L2/[], M2/[c, d, e], N1/[A2|N2]}

```
:- concat([], [c, d, e], N2)
```

C1

σ_3 = {N2=M3=[c, d, e]}

□

R = [A1|N1] = [a|[A2|N2]] = [a | [b | [c, d, e]]] = [a, b, c, d, e]